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# D-180219

## B. Tech. EXAMINATION, 2018

Semester I (CBS)

ENGINEERING MATHEMATICS-I (A & B)

MA-101

*Time : 3 Hours* *Maximum Marks : 60*

*The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.*

**Note :** Attempt *Five* questions in all, selecting *one* question from each Sections A, B, C and D. Section E is compulsory.

### Section A

**1.** (a) Find the values of $a$ and $b$ for which the equations $x + ay + z = 3$, $x + 2y + 2z = b$, $x + 5y + 2z = 9$ are consistent. When will these equations have a unique solution ? **6**





(b) Find the eigen values and eign vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **6**

**2.** State and prove Cayley Hamilton Theorem. Verify this theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Hence find $A^{-1}$. **12**

### Section B

**3.** (a) Solve the equation $x^7 + x^4 + x^3 + 1 = 0$ by complex root method. **6**

(b) If $u = \log \tan\left(\frac{u}{4} + \frac{\theta}{2}\right)$, then prove that

$$\tanh \frac{u}{2} = \tan \frac{\theta}{2}.$$ **6**

**4.** (a) Separate real and imaginary parts of $\sin^{-1}(\cos\theta + i\sin\theta), 0 < \theta < \frac{\pi}{2}$. **6**

(b) Sum the series :

$$1 + x\cos\alpha + x^2\cos 2\alpha + x^3\cos 3\alpha + \ldots\ldots\ldots n \text{ terms}$$

where $x$ is less than unity. also find the sum to infinity. 6

## Section C

5. (a) If $u = f(r)$ where $r^2 = x^2 + y^2 + z^2$, then prove that : 6

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f''(r) + \frac{2}{r} f'(r)$$

(b) Examine for maximum and minimum values of $\sin x + \sin y + \sin(x+y)$. 6

6. (a) Evaluate $\iint_R y\, dxdy$, where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. 6

(b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. (Use double integration). 6





## Section D

7. (a) Find the directional derivative of the function $2xy + z^2$ at the point (1, –1, 3) in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$. 6

(b) Show the vector field $\bar{A}$, where :

$$\bar{A} = (y\sin z - \sin x)\hat{i} + (x\sin z + 2yz)\hat{j} + (xy\cos z + y^2)\hat{k}$$

is irrotational. Find the scalar potential $u$ such that $\bar{A} = \text{grad}\, u$. 6

8. (a) Use Green's theorem to evaluate the line integral $\oint_S \left((xy^2)dx + x^2 dy\right)$, where C is the boundary of the closed region bounded by $y = x,\ y = x^2$. 6

(b) Use the divergence theorem to evaluate $\iint_S (xdydz + ydzdx + zdxdy)$, where S is the portion of the plane $x + 2y + 3z = 6$ lies in the first octant. 6

## Section E

9. Attempt all the questions : $1\times12=12$

(a) If $\lambda$ be an eigen value of a non-singular matrix A, then show that $\lambda^{-1}$ is an eigen values of $A^{-1}$.

(b) Sum of eigen values of a matrix is............

(c) Prove that $\sin z$ is a periodic function, where $z$ is a complex function.

(d) Find the general value of $\log(1+i)$, where $i=\sqrt{-1}$.

(e) Separate real and imaginary parts of $\sin(x+iy)$.

(f) Find the rank of the matrix :

$$A=\begin{bmatrix}1 & 2 & 3 & 4\\ -2 & 0 & 5 & 7\end{bmatrix}$$

(g) Prove that $\Gamma(n+1)=n!; n>0$ and $\Gamma$ is Gamma function.

(h) Evaluate the integral :

$$\int_0^1\int_{y^2}^1\int_0^{1-x} x\,dz\,dx\,dy$$

(i) Find the first order partial derivatives of $\cos^{-1}\left(\frac{x}{y}\right)$.

(j) If $r=|\vec{r}|$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, then show that

$$grad\left(\frac{1}{r}\right)=-\frac{\vec{r}}{r^3}.$$

(k) Prove that :

$$\text{curl}(\text{grad}\,\phi)=0$$

(l) State Stoke's theorem.